

Torque



→ Tangential forces to rotational motion.

$$F_T = m a_T$$

$$a_c = r\omega^2 \quad a_T = r\alpha \quad v = r\omega$$

$$\omega = \frac{2\pi}{T} \text{ or } \frac{d\theta}{dt} \quad \frac{d^2\theta}{dt^2}$$

$$\text{Torque} = \tau = F_T r = m a_T r = m r^2 \alpha$$

Treat rigid objects as sum of all the individual particles

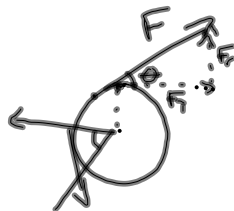
$$\tau_{\text{net}} = \sum m_i r_i^2 \alpha$$

$$\tau_{\text{net}} = \left(\sum m_i r_i^2 \right) \alpha = I \alpha$$

$$\tau_{\text{net, ext}} = I \alpha$$

Newton's second Law for torque.

think torque_{net, ext} = I α



$$F_T = F \sin \phi$$

Torque - Moment of Force

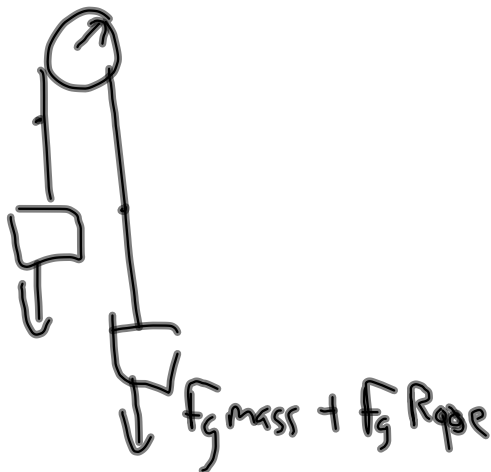
$$\tau = F_T r = F \sin \phi r = F \ell$$

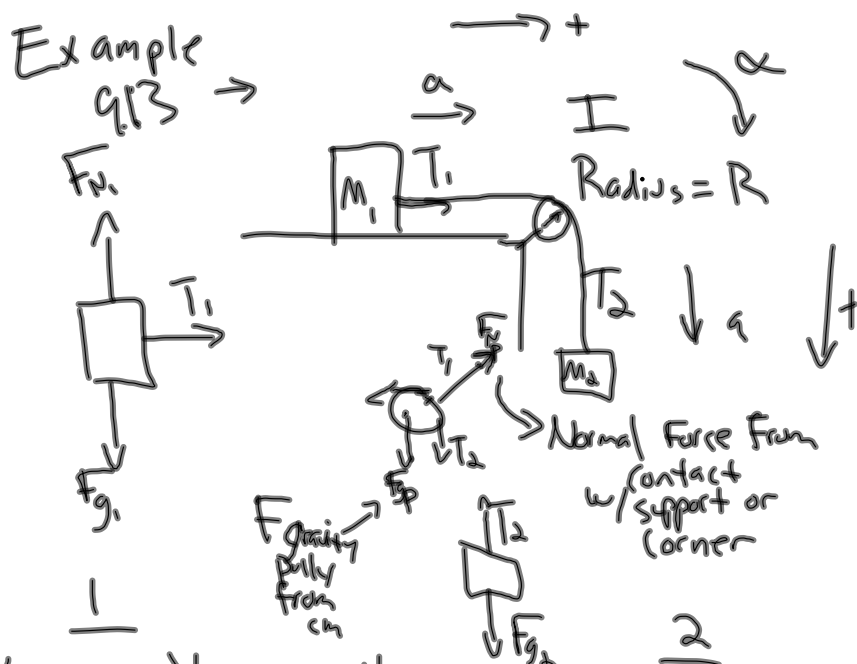
↑
length of moment arm (lever)

Non Slip - pulley systems

→ rope or string going around pulley has the same tangential velocity as pulley itself

$$v_t = R\omega = R = \text{radius of pulley}$$





$\frac{x}{T_1}$	$\frac{y}{T_2}$	putley rotational	$\frac{x}{T_2}$	$\frac{y}{F_{g2}}$
$\frac{m_1 a}{0}$	$\frac{-T_2}{m_2 a}$	$\frac{-T_1 R}{T_2 R}$	$\frac{a, T_1 + T_2}{I \alpha}$	$\frac{-T_2}{m_2 a}$

$T_1 = m_1 a$ $I \alpha$ $m_2 g - T_2 = m_2 a$

$T_2 R - T_1 R = I \alpha$ $T_2 = m_2 (g - a)$

Non-slip $\leftarrow \alpha = \frac{a_T}{r}$
 $a_T = a$ $r = R$ $a_T = r \alpha$

$T_2 R - T_1 R = \frac{I a}{R}$

$m_2 (g - a) R - m_1 a R = \frac{I a}{R}$

$m_2 g R - m_2 a R - m_1 a R = \frac{I a}{R}$

$m_2 g R = \left(\frac{I + m_2 R^2 + m_1 R^2}{R} \right) a \quad \therefore a$

$a = \frac{m_2 g R^2}{I + m_2 R^2 + m_1 R^2}$