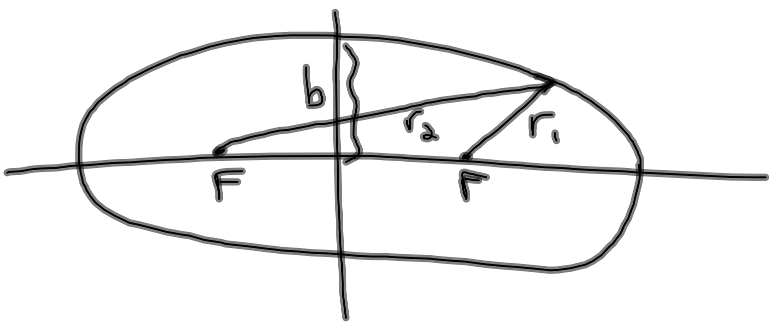
Gravity

Kepler's Laws (3)

1) All planets move in an elliptical orbit with the sun at one of the foci



For planets -

perihelion → point closest to the sun

aphelion → point furthest from the sun

semimajor = Average distance from sun

For earth

1.5×10^{11} m = 1 A.U. = (1 amu)

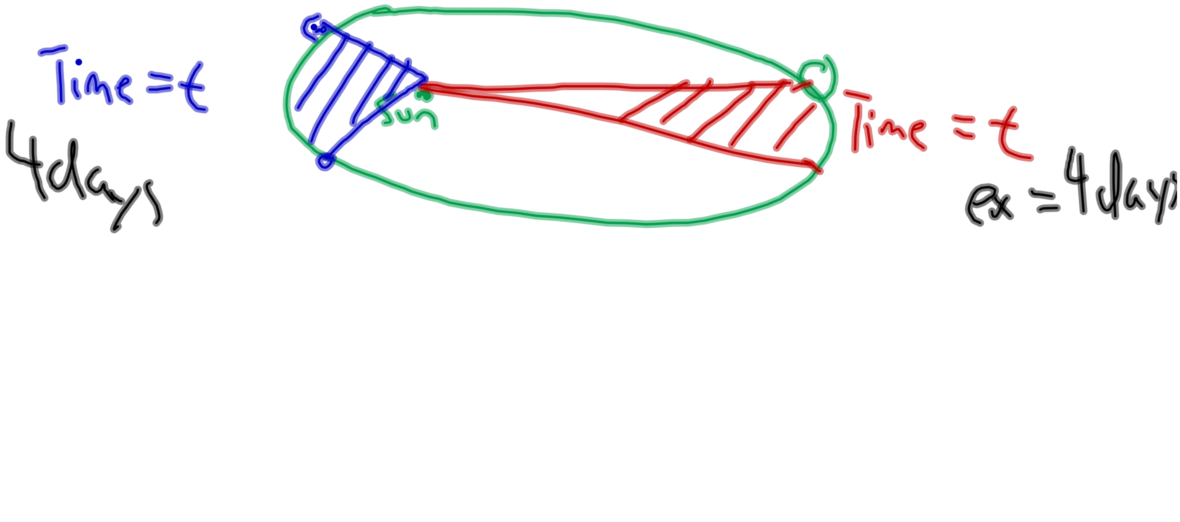
93.0×10^{6} Miles

Astronomical Unit

astronomical mean unit

Keplers 2^{nd} Law -

The area swept out between a planet and the sun during its orbit is always equal for equal times.



3^{rd} Law

The square of the period of any planet is proportional the cube of the semimajor axis.

$$T^2 = Cr^3$$

C = constant for all planets

$$T^2 = Cr^3$$

For Neptune $164.8^2_{yrs} = Cr^3$

For $\oplus$ $1yr^2 = Cr^3$

$1yr^2 = C\,1AU^3$

$$\frac{164.8^2}{1^2} = \frac{r^3}{1^3}$$

$$164.8^2 = r^3$$

$$r = 164.8^{2/3}$$

$r = 30.06$ A.U.

4.5×10^{12} m

$r_e = 6371$ km $= 6.37 \times 10^6$ m

$M_e = 5.98 \times 10^{24}$ kg

Universal Law of gravitation -

- Force due to gravity from object 1 on object 2 is

$$\vec{F} = -\frac{G m_1 m_2}{r_{12}^2}\hat{r}_{12}$$

r_{12} = distance between 1 and 2

$\hat{r}_{12}$ = unit vector to indicated direction

G = universal gravitation constant

$= 6.67 \times 10^{-11}$ Nm^2/kg^2

$F = G\frac{m_1 m_2}{r^2}$ ← magnitude

you as m_1

Earth as m_2

$$\cancel{m_1} a = \frac{\left(6.67 \times 10^{-11} \frac{Nm^2}{kg^2}\right) \cancel{m_1}\, 5.98 \times 10^{24} kg}{(6.4 \times 10^6)^2} =$$

For near circular orbits special case of kepler's s Laws.

$$F = \frac{G m_1 m_2}{r^2}$$

m_1 = Mass Sun $= M_s$ m_2 = Mass of planet in "circular" orbit

$$F = \frac{G M_s \cancel{M_p}}{r^{\cancel{2}}} = M_p a_p = \cancel{M_p} \frac{v^2}{\cancel{r}}$$

$$v^2 = \frac{G M_s}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$v = \frac{2\pi r}{T} \qquad T^2 = \frac{4\pi^2 r^3}{G M_s}$$

$$T^2 = \left(\frac{4\pi^2}{G M_s}\right) r^3$$

for circular orbits, $C = \dfrac{4\pi^2}{G M_s}$

- used for sattelites orbiting planets. (natural (ie. moons) + others)

M_s would be mass of planet

$$dU = -F \cdot dl$$

$$dU = -\vec{F_g} \cdot dl = -(-F_g \hat{r})\, dl$$

$$dU = \frac{GM_e m}{r^2}\, dr = -\frac{U(r)}{r}\, dr$$

$$U = GM_e m \int r^{-2}\, dr$$

$$= -r^{-1} + \cancel{C} \rightarrow U_0$$

assuming $U_0 = 0$ for reference level

$$U = -\frac{GM_e m}{r}$$

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escape velocity

$$V_e = \sqrt{\frac{2GM_E}{R_E}}$$

M_e, R_e for earth

apply for whatever planet look

$$6.67 \times 10^{-11}\ Nm^2/kg^2$$

Gravitational Fields

$F = ma \qquad a = g(r)$

$F_g = m g(r)$

$$g(r) = \frac{F_g}{m} = \frac{G m m_2}{r^2 m}$$

$$g(r) = \frac{G m_2}{r^2}$$ $\leftarrow$ Mass of planet

→ Gravitational fields.