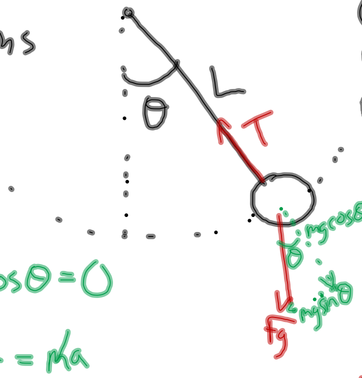
Pendulums



Can be modeled using SHM But only for small $\theta \ll 1$ rad oscillation angles.

$T - mg\cos\theta = 0$

$-mg\sin\theta = ma$

Restoring force

$a = -g\sin\theta$

Normally

$a = \frac{d^2x}{dt^2}$

now since an arc $a = \frac{d^2s}{dt^2}$

$$L\frac{d^2\theta}{dt^2} = -g\sin\theta$$

or

$$\frac{d^2\theta}{dt^2} = \frac{-g}{L}\sin\theta$$

$s = R\theta$

$s = L\theta$

$\frac{d^2s}{dt^2} = L\frac{d^2\theta}{dt^2}$

for $\theta \ll 1$ $\sin\theta \sim \theta$

$$\frac{d^2\theta}{dt^2} \approx \frac{-g}{L}\theta$$

$$\omega = \sqrt{\frac{g}{L}}$$

$\omega^2 = \frac{k}{m}$

$F = -kx$

ma

$\Rightarrow a = \frac{-k}{m}x = -\omega^2 x$

For pendulums

$\left(T = \frac{2\pi}{\omega}\right)$ SHM

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\theta = \theta_0\cos(\omega t + \delta)$$

For $\theta \ll 1$

For θ larger

$$T = T_0\left[1 + \frac{1}{2^2}\sin^2\left(\frac{1}{2}\theta_0\right) + \frac{1}{2^2}\left(\frac{3}{4}\right)^2\sin^4\left(\frac{1}{2}\theta_0\right) + \ldots\right]$$

$\theta \approx 5\text{-}15°$

Physical Pendulum

Some mass oscillating like a pendulum about some axis

$$T = 2\pi\sqrt{\frac{I}{MgD}}$$

$I = I_{cm} + MD^2$

I = Moment of inertia, D = Distance to axis of rotation from c.m.

Damping

- Overdamping - doesn't even complete 1 Oscillation
- underdamping - small damping effects over several oscillations, (small changes in A or θ_0)
- Critically damped - "just above" underdamped → so leads to non oscillation behavior.