Simple Harmonic Motion, Springs, Pendulums & Hooke's Law

Define - SHM

→ oscillatory motion due to a restoring force that is directly proportional to the displacement.

- Springs & Pendulums are 2 most common examples

Spring - Hooke's Law

$$F = -kx$$

k = spring constant $\left(\frac{N}{m}\right)$
- specific for each spring.

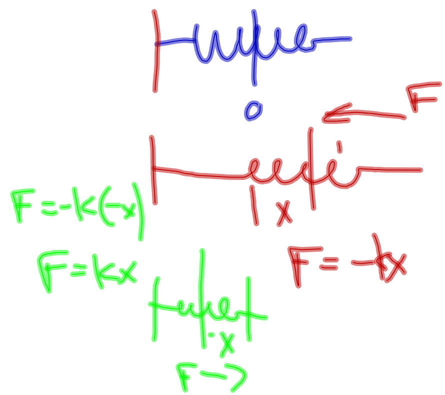
x = displacement from equilibrium

− ← | → +

compressing | stretching

eq.

The Force always points towards equilibrium position (restore)



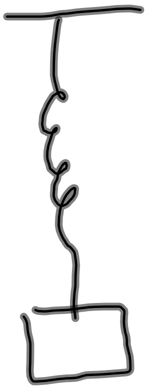
Period = T = Time for 1 complete oscillation (s)

Frequency = f = $\frac{\text{How many oscillations}}{\text{second}}$ = $\frac{1}{T}$ = Hz (Hertz)

X_{max} = Amplitude = A = Max displacement from equilibrium (m)

$F = -kx$ = Hooke's Law

Energy



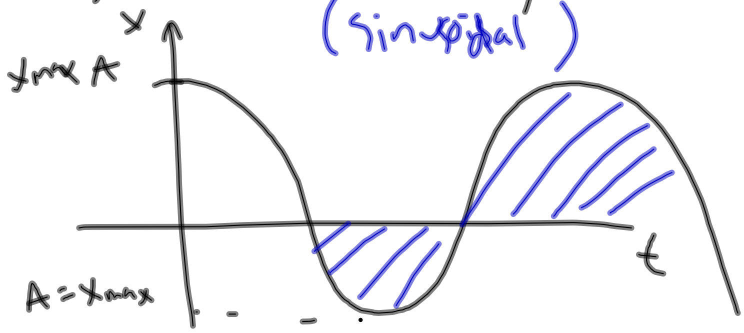
Elastic or Spring Potential

$\uparrow F_s$ $\quad U_s$ or $PE_s = \frac{1}{2}kx^2$

k = spring constant

$\downarrow F_g$ $\quad$ & x = disp. from equilibrium

SHM - Graphically (sinusoidal)



neglecting friction, heat, etc.

$$x = A\sin(\omega t)$$

ω = angular speed = rad/sec

$$f = \frac{1}{T} \qquad V = \frac{2\pi x}{T}$$

(wave speed or velocity of vibration)

Model as $a = \frac{v^2}{x}$ $\qquad V = \frac{2\pi x}{T}$

$$F = -kx$$

$$ma = -kx$$

$$m\frac{v^2}{x} = -kx$$

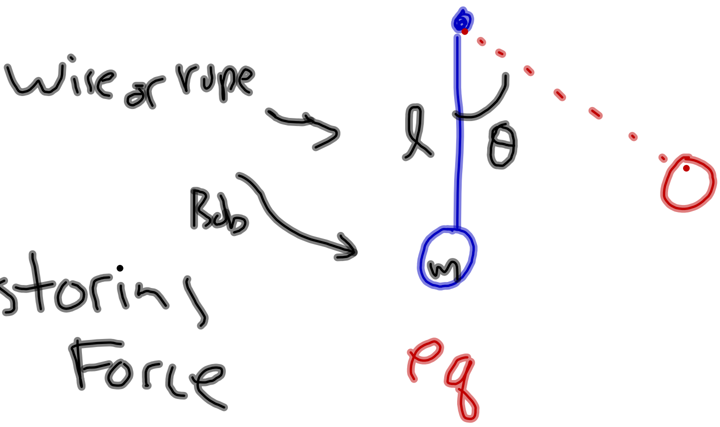
$$T^2 \cdot x \cdot \frac{m \cdot 4\pi^2 x^2}{T^2 \cdot x} = -kx \cdot x \cdot T^2$$

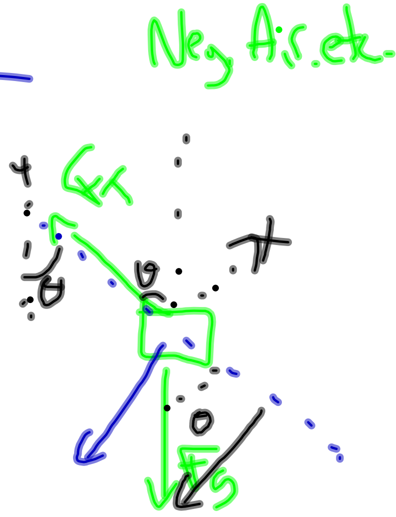
$$m \cdot 4\pi^2 \cancel{x^2} = \cancel{-}kT^2 \cancel{x^2}$$

$$T^2 = \frac{m \cdot 4\pi^2}{k} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

$$f^2 = \frac{k}{m 4\pi^2} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Pendulums





Restoring Force

$= -F_g \sin\theta$

$= -mg\sin\theta$

X	Y
$-F_g \sin\theta$	$-F_g \cos\theta$
	F_T
	0

Pendulums only obey SHM for small θ $\simeq < 15°$

can act like it to 40° sometimes.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

l = length
g = gravity