Specific Heat

- heat added or subtracted per unit change in Temp.
- constant only specific to the process

$$c = \frac{\delta q}{dT}$$

$$C_v = \left(\frac{\delta q}{dT}\right)_{\text{constant volume}}$$

$$C_p = \left(\frac{\delta q}{dT}\right)_{\text{constant pressure}}$$

$$\delta q = C_v dT$$

$$\delta q = C_p dT$$

Constant Volume Process

$$\delta q = de + \cancel{pdv}^{\,0}$$

$$de = \delta q = C_v dT$$

$$\int_0^e de = \int_0^T C_v dT$$

$$e = C_v T$$

Assuming at $T = 0$, $e = 0$ & $C_v = \text{const}$

For Constant Pressure

$$\delta q = dh - \cancel{vdp}^{\,0}$$

$$\delta q = \int_0^h dh = \int_0^T C_p dT$$

$$h = C_p T$$

Assuming @ $T = 0$, $h = 0$, $C_p = \text{const.}$

These hold for any perfect gas, and any process.

$$e = C_v T$$

$$h = C_p T$$

pg 151
eq 4.6

For air @ standard sea-Level

$C_v = 720 \frac{J}{KgK}$

$= 429 \frac{ft\,lb}{slug°R}$

@ $T = 288.16K$

$= 519°R$

$C_p = 1008 \frac{J}{KgK}$

$= 6006 \frac{ft\,lb}{slug°R}$

$e = C_v T$

$h = C_p T$

1. Constant volume
2. Constant pressure.
3. <u>Adiabatic</u> $\Rightarrow \delta q = 0$

   → No heat added or taken out
4. <u>Reversible</u> - No friction or other external (dissipative) effects.
5. <u>Isentropic</u> - BOTH

   Adiabatic + Reversible

- Look at flow over an airfoil

→ No external sources of heat or heat exchange.

*Note: No heat added or taken out does NOT mean No Temp Change! *

ie. Change in volume
- compressible
  → work

ie. Constant volume
- incompressible

  No work, No Temp. change.

Assuming an Adiabatic process

$$\delta q = 0$$

$$de = \cancel{\delta q}^{\to 0} - p\,dv = C_v\,dT$$

$$dh = \cancel{\delta q}^{\to 0} + v\,dp = C_p\,dT$$

$$\frac{de}{dh} = \frac{-p\,dv}{v\,dp} = \frac{C_v}{C_p}$$

$$\frac{dv}{v} = -\frac{dp}{p}\frac{C_v}{C_p}$$

or

$$\frac{dp}{p} = -\frac{dv}{v}\frac{C_p}{C_v}$$

$$\frac{C_p}{C_v} = \gamma \quad = \text{ratio of specific heat}$$

1.4 for air

$$\int_{P_1}^{P_2} \frac{dp}{p} = -\gamma \int_{v_1}^{v_2} \frac{dv}{v}$$

Look at pts in an isentropic flow

$$\ln p \Big|_{P_1}^{P_2} = -\gamma \ln v \Big|_{v_1}^{v_2}$$

$$\ln P_2 - \ln P_1 = -\gamma\,(\ln v_2 - \ln v_1)$$

$$\ln \frac{P_2}{P_1} = -\gamma \ln \frac{v_2}{v_1}$$

$$e^{\cancel{\ln}} \frac{P_2}{P_1} = \cancel{e}^{\cancel{\ln}} \left(\frac{v_2}{v_1}\right)^{-\gamma} \qquad \left(\frac{2}{1}\right)^{-1} = \frac{1}{2}$$

$$\frac{P_2}{P_1} = \left(\frac{v_2}{v_1}\right)^{-\gamma} = \left(\frac{v_1}{v_2}\right)^{\gamma}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

if v = Sp. Vol

$$p = \cancel{\rho} RT$$

$$\frac{1}{\rho} \quad \frac{\cancel{\rho}}{\cancel{\rho}}$$

$$PV = RT$$

$$V_1 = \frac{RT_1}{P_1}$$

$$V_2 = \frac{RT_2}{P_2}$$

$$\frac{P_2}{P_1} = \left(\frac{\frac{\cancel{R}T_1}{P_1}}{\frac{\cancel{R}T_2}{P_2}}\right)^{\gamma}$$

$$(6x)^2 = 6^2 \cdot x^2$$

$$\frac{P_2}{P_1} = \left(\frac{T_1 P_2}{T_2 P_1}\right)^{\gamma} = \left(\frac{T_1}{T_2}\right)^{\gamma}\left(\frac{P_2}{P_1}\right)^{\gamma}$$

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$$\left(\left(\frac{P_2}{P_1}\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}} = \left(\left(\frac{T_1}{T_2}\right)^{\gamma}\right)^{\frac{1}{1-\gamma}}$$

$$\left(6^2\right)^{1/3} = \left(x^3\right)^{1/3}$$

$$x = 6^{2/3}$$

$$\frac{P_2}{P_1} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{1-\gamma}}$$

$$\boxed{\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{-\frac{\gamma}{1-\gamma}} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

Isentropic Compressible Flow

<u>Incompressible</u>	<u>Compressible</u>
Bernoulli	Euler
Continuity $= A_1\vec{V}_1 = A_2\vec{V}_2$ (V = Velocity)	$\rho_1 A_1 \vec{V}_1 = \rho_2 A_2 \vec{V}_2$
	Isentropic Flow eq.

Ex 4.7 Sea level!

$T_1 = 250\,K$

$p_1 = ?$

$p_\infty = 1.01325 \cdot 10^5 \frac{N}{m^2}$

$T_\infty = 288.16\,K$

$$\frac{p_1}{p_\infty} = \left(\frac{T_1}{T_\infty}\right)^{\frac{\gamma}{\gamma-1}} \quad \gamma = 1.4$$

$$p_1 = 61{,}400 \frac{N}{m^2} \frac{ft\,ft}{ft\,ft}$$